ASSIGNMENT SET - I

Department of Mathematics

Mugberia Gangadhar Mahavidyalaya

B.Sc Hon.(CBCS)

Mathematics: Semester-V

Paper Code: C12T

[Group Theory-2]

Answer all the questions

Unit-1

1. Define the terms automorphism and inner automorphism for a group. Show that the inner automorphism of group G form a normal subgroup of the automorphisim group Aut(G). 2+4
2. Suppose that φ is an isomorphism from a group G onto a group $\bar{G}$. Then prove that $\Phi\big(Z(G)\big) = Z(\bar{G})$. 3
3. Let G be a group, then show that $f: G \to G$ defined by $f(x) = x^{-1}$ is an automorphism of G iff G is abelian. 4
4. Prove that every characteristic subgroup of G is normal subgroup.2
5. Prove that the centre Z=Z(G) of G is a characteristic subgroup. 3

Unit-II

1. Let $G = Z_{10} \times Z_{10}$, then show that G contains exactly 24 elements of order5 and contains exactly 24 elements of order 10. 2+2

2. Let G be a group of order 77. Then show that centre of G, Z(G) is isomorphic to Z_{77}. 3
3. Find all abelian groups up to isomorphism of order 360. 2
4. Let p be prime and m, a positive integer such that p^m divides o(G). Then there exist a subgroup H of G such that $o(H) = p^m$.

Unit-III

1. Prove that the only group of order 255 is Z_{255}.
2. Let G be a group of order pqr, p$p < q < r$ being prime. Prove that
   (i) Sylow r subgroup is normal in G.

   (ii) G has a normal subgroup of order qr.

   (iii) If q does not divide (r-1) then Sylow q-subgroup is normal.

3. Show that there is no simple group of order 216.
4. Let o(G)=30. Show that.
   (i) Either Sylow 3-subgroup or sylow 5-subgroup is normal in G.
   (ii) G has a normal subgroup of order 15.
   (iii) Both sylow 3-subgroup and sylow 5-subgroup are normal in G.

Unit-IV

1. Let G be a finite group acting on a finite set S. Let $x_1, x_2, \dots, x_n \epsilon S$ be such that $Gx_i \neq \{x_i\}$ for any i=1,2,3,…,n. then

$$o(S) = o(F_s) + \sum_{i=1}^{n} [G : Gx_i]$$

where$[G : Gx_i]$ denotes the index of Gx_i in G.

2. Let G be a non abelian group of order p^3. Determine o(Z(G)) and k= number of conjugate classes of G.

3. Let $A = \{1,2,3\}$ and $G = S_3$. Define $*: G \times A \to A$ such that $\sigma * a = \sigma(a)$. Show that * is a group action and find all the stabilizers and orbits. Is the action transitive?
4. Let G be group and Z(G) be the center of G. Then show that $\frac{G}{Z(G)} \cong I(G)$ where I(G) is the set of all inner automorphism of G.

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